



POLITECNICO
MILANO 1863



Advanced Modeling for Operations

Project Work

Part 1

Group 10

Alessio Cerboni 10934057

Luca Panti 10707591

Ludovica Caroni 10665249

Marco Martinelli 10681868

Tushar Srikumar Nair 10890787



Agenda

- Introduction
- Base Case Performances
- Ordering & Inventory Policies
- Demand Growth Estimation
- Cost Analysis
- Pre-Simulation
- Panic Buying Dynamics
- Customer Movement
- Distribution Center Waitlist
- Modified Model Performance
- Business insights
- Appendix

Introduction

Supply Chain Dynamics under Panic Buying

The aim of the project is to **analyze** and **improve** the dynamics of grocery supply chains, particularly focusing on pasta, in response to panic buying during large-scale disasters like a pandemic.



Key Highlights of the Current Model

Network Structure

The supply chain is modeled as a network of manufacturers, distribution centers, stores, and customers. This structure captures the flow of pasta products from production to consumption across different regions.

Inventory Management

The simulation includes a 'Make-To-Stock' production process and 'Pull' logic in inventory control. It reflects real-world supply chain operations without safety stocks, where production capacity meets distribution center demands and restocking occurs periodically.

Purchasing Process

Customer behavior is modeled with varying purchasing frequencies and quantities, capturing realistic consumption patterns. The process also includes scenarios where customers change stores if desired quantities are unavailable.

Panic Buying Dynamics

The model triggers panic buying in response to a pandemic, doubling the purchasing frequency and quantity of affected customers. This sudden change provides insights into supply chain resilience under stress.

Base Case Performance

To achieve a comprehensive understanding of the base case performance and identify areas for potential improvement, some KPIs of the model were plotted.

Initialization Parameters

- No. of Manufacturers = 1
- No. of Distribution Centers = 1
- No. of Stores = 100
- No. of Customers = 30000
- No. of Runs = 1
- No. of days in a run = 100
- Pandemic Start = Day 15
- Panic End = Day 35

Demand Variation over Time

- A spike in demand is visible right after the pandemic starts, signifying an initial panic buying reaction.
- The demand appears to settle down but remains above the pre-pandemic level throughout the panic period, suggesting sustained increased demand.
- After the panic ends, the demand seems to decrease but does not return to the original levels within the observed timeframe.

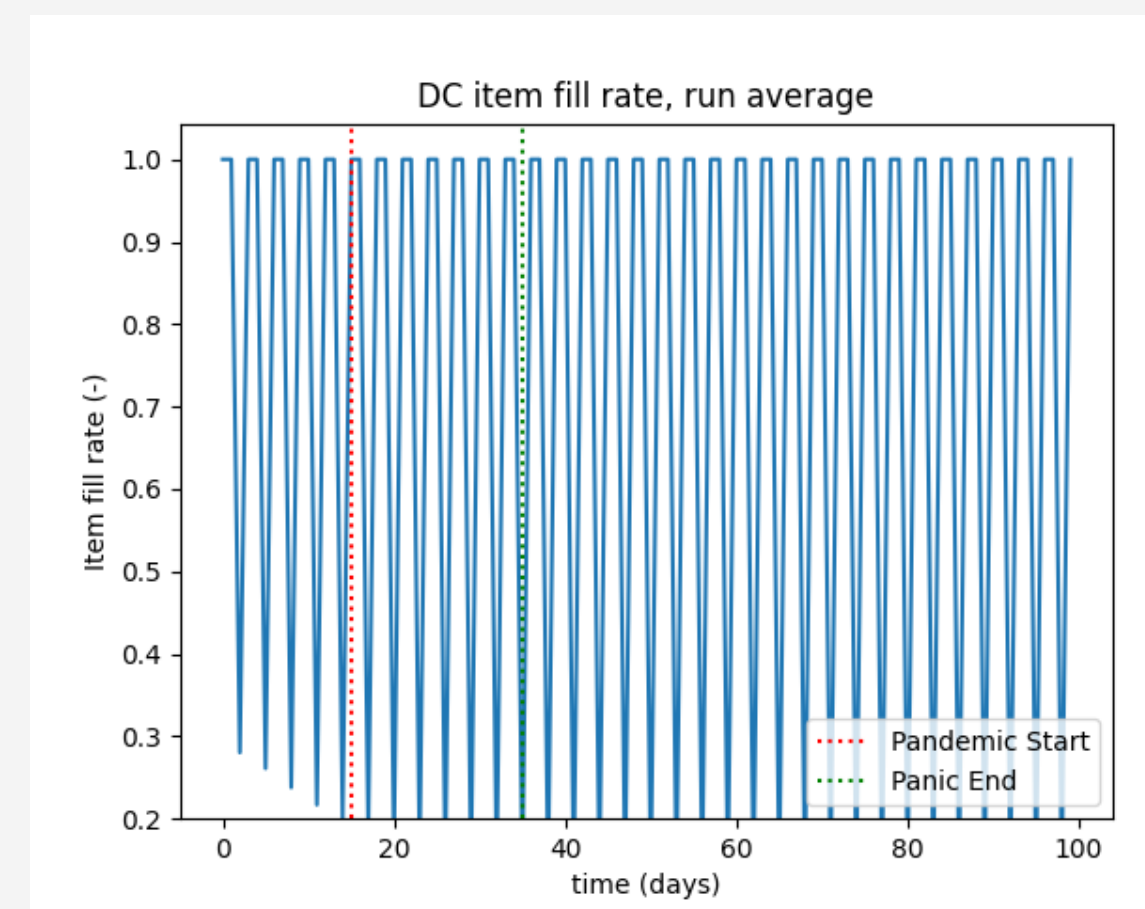
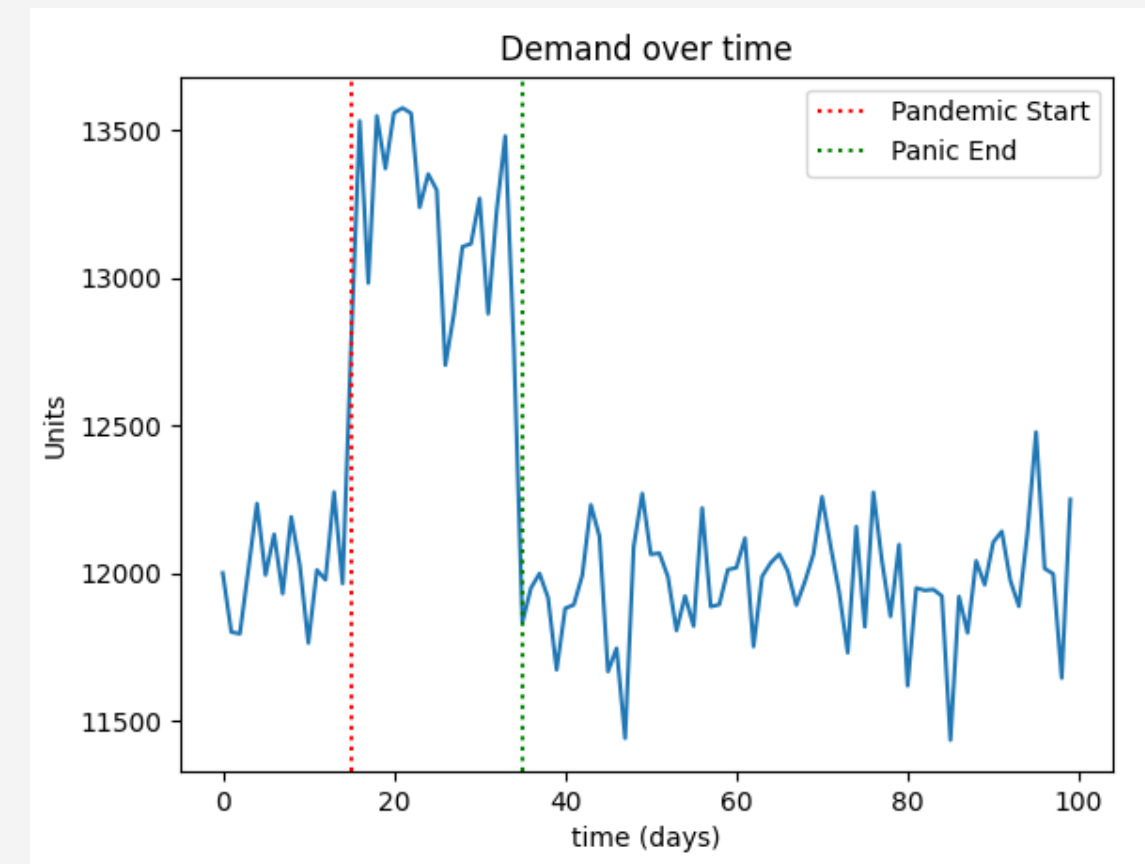
Item Fill Rates

$$ITEM\ FILL\ RATE_{DC} = \frac{1 - LOST\ DEMAND}{TOTAL\ DEMAND}$$

Minimum item fill rate in normal period: **12.74 %**

Minimum item fill rate during pandemic: **13.44 %**

Both percentages are relatively low, indicating that a significant portion of demand is unmet both during normal times and the pandemic.



Ordering & Inventory Policy

Base Case

Safety Stock

Neither distribution centers nor stores maintain safety stocks, implying a lean inventory strategy with the potential risk of stockouts during unexpected demand surges.

Restocking Policy and Demand Estimation

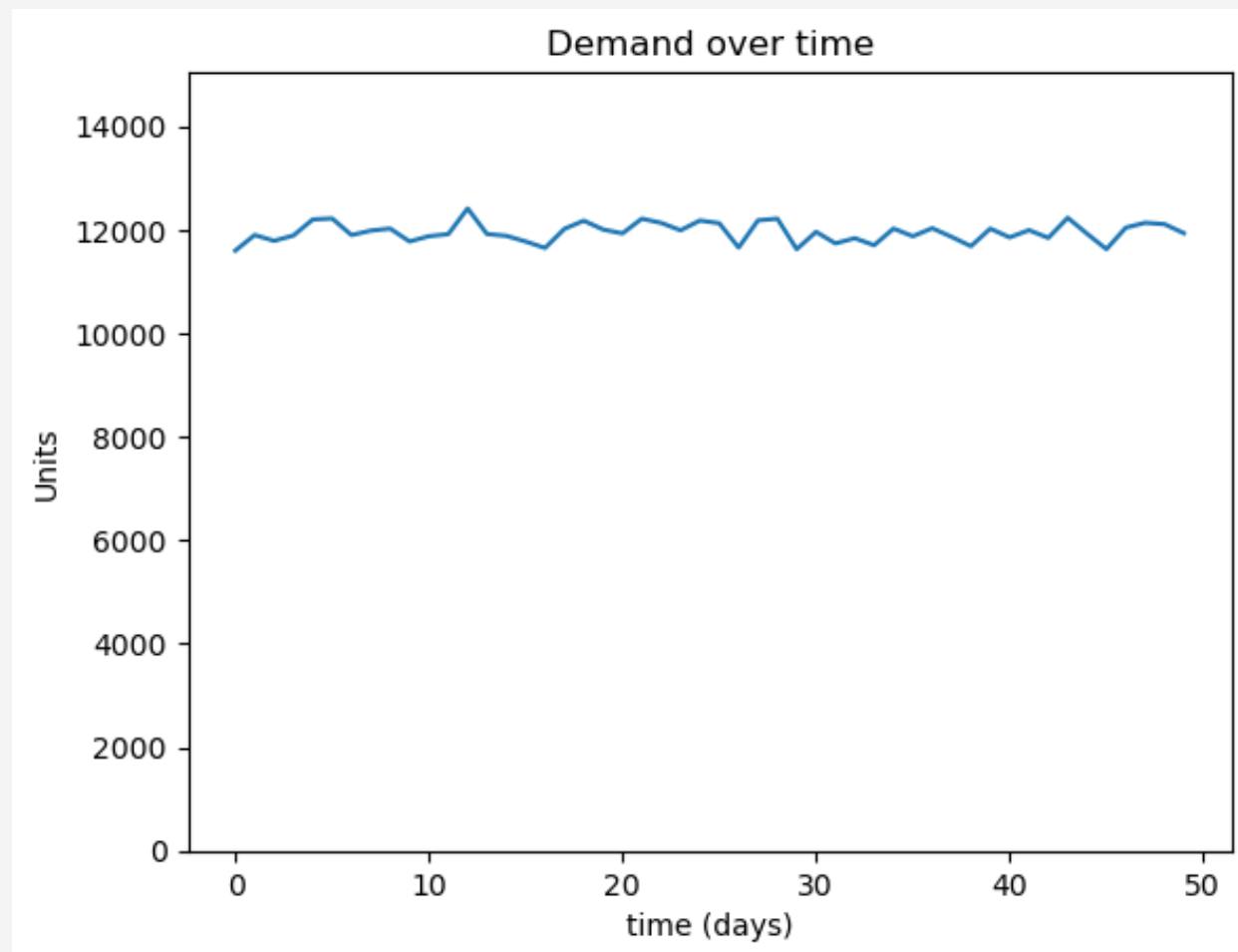
- The distribution center's daily demand was calculated using a 3-day average of restocking data, while store demand was derived from a 7-day observation period. Only the days with nonzero demand were included in these averages.
- Nevertheless, this method incorporates days when the inventory depletes by day's end, potentially resulting in an underestimated demand.
- This could occur on days when actual demand exceeds supply, leading customers to encounter stock shortages. This was changed in the new version of the code, excluding also these particular days.

Inventory cost

- The demand data from the past shows no discernible trend or seasonality, indicating a remarkably stable demand pattern. Ironically, this repetitive and seemingly unnecessary effort of forecasting, which lacks any substantial change in the data to justify it, often results in increased costs.
- This routine of frequent forecasting, in the absence of any significant variability in demand, seems to be an inefficient use of resources, leading to higher operational costs without yielding any benefits.

Policy (R,S)

- The **Fixed Period Reorder Point (R)** and **Target Availability inventory (S)** management policy is used to streamline and optimize stock levels in a business.
- Under this approach, inventory is reviewed and restocked at fixed intervals (e.g., weekly, monthly) named R, ensuring a consistent replenishment cycle.
- The target availability level is set based on desired service levels (minimum item fill rate in our case) and demand variability, aiming to maintain enough stock to meet anticipated demand while minimizing excess inventory.



- In our scenario, the demand for pasta exhibits remarkable stability, with minimal variability when compared to the average demand levels.
- This consistent demand pattern renders the continuous prediction of future orders based on past data an unnecessary exercise that incurs additional effort and cost.
- Consequently, the (R,S) policy, with its predetermined reorder points and levels, emerges as the most suitable approach for inventory management in this context.

Policy (R,S) - Design Parameters

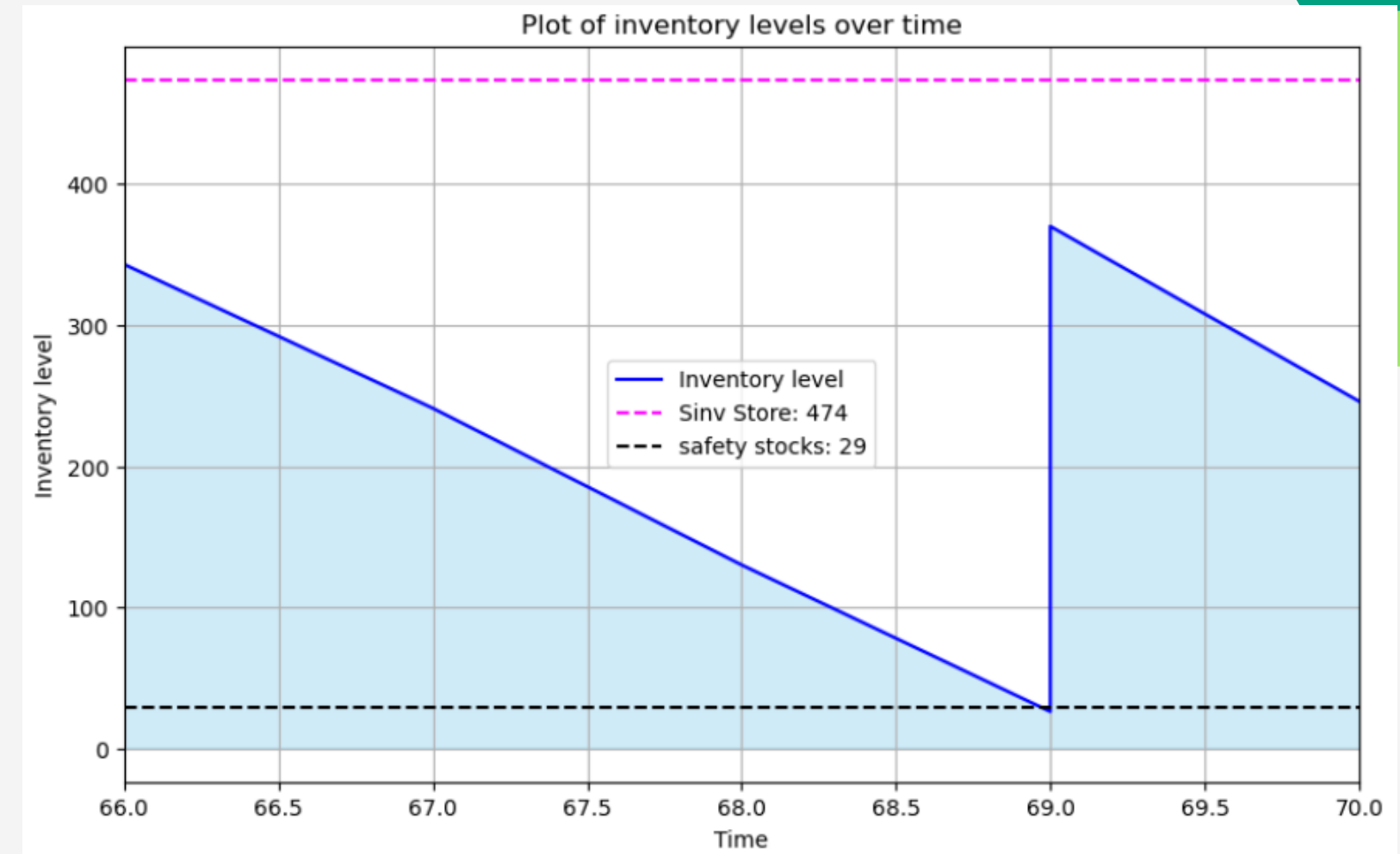
The policy was developed in two distinct variants: one tailored for the Stores and another for the Distribution Center (DC) and Manufacturer. The key to achieving the appropriate levels of safety stocks and thus the desired **Target Inventory Availability (S)** lay in adopting a function that established a minimum service level, specifically a minimum **Item Fill Rate**. This approach began with the formulation of the item fill rate:

$$IFR = P_2 = 1 - \frac{Lost\ Demand}{Demand} \stackrel{*}{=} 1 - \frac{\sigma_L \times G_u(k)}{Demand}$$

$$\Rightarrow G_u(k) = \frac{Demand \times R}{\sigma_{R+L}} \times \frac{1 - P_2}{P_2} \stackrel{*}{\Rightarrow} k$$

*Look at the Appendix to see the demonstration.

Variable name	Symbol
Safety stocks	ss
Safety factor	k
Fixed Period Reorder Point	R
Target Availability inventory	S
Gu(k)	It is a parameter used to esteem the Safety factor k, to guarantee a minimum value of item fill rate P2



To determine the appropriate level of Safety Stocks needed to achieve the desired target service level, we calculate the Safety Factor (k). This factor is integral in designing the inventory to meet specific service objectives.

$$ss = k \times \sigma_{R+L}$$

$$S = Demand\ During\ Cycle + ss$$

Policy (R,S) - Service Level

For the distribution center and the manufacturer, a somewhat different method was employed to calculate the target function. Rather than utilizing the $G_u(k)$ to determine the safety stock, the Cycle Service Level, also known as $P_u(k)$, was applied. The $P_u(k)$ represents the likelihood of avoiding a stockout over a replenishment cycle, which is equivalent to one minus the probability of experiencing a stockout:

$$P_u(k) = 1 - P_1 \xrightarrow{*} k$$

**Look at the Appendix to see the demonstration.*

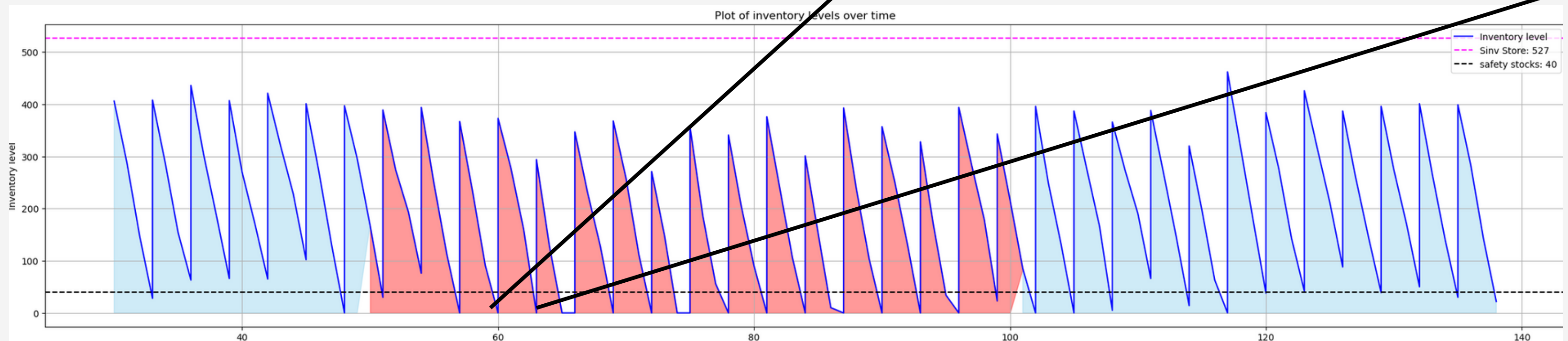
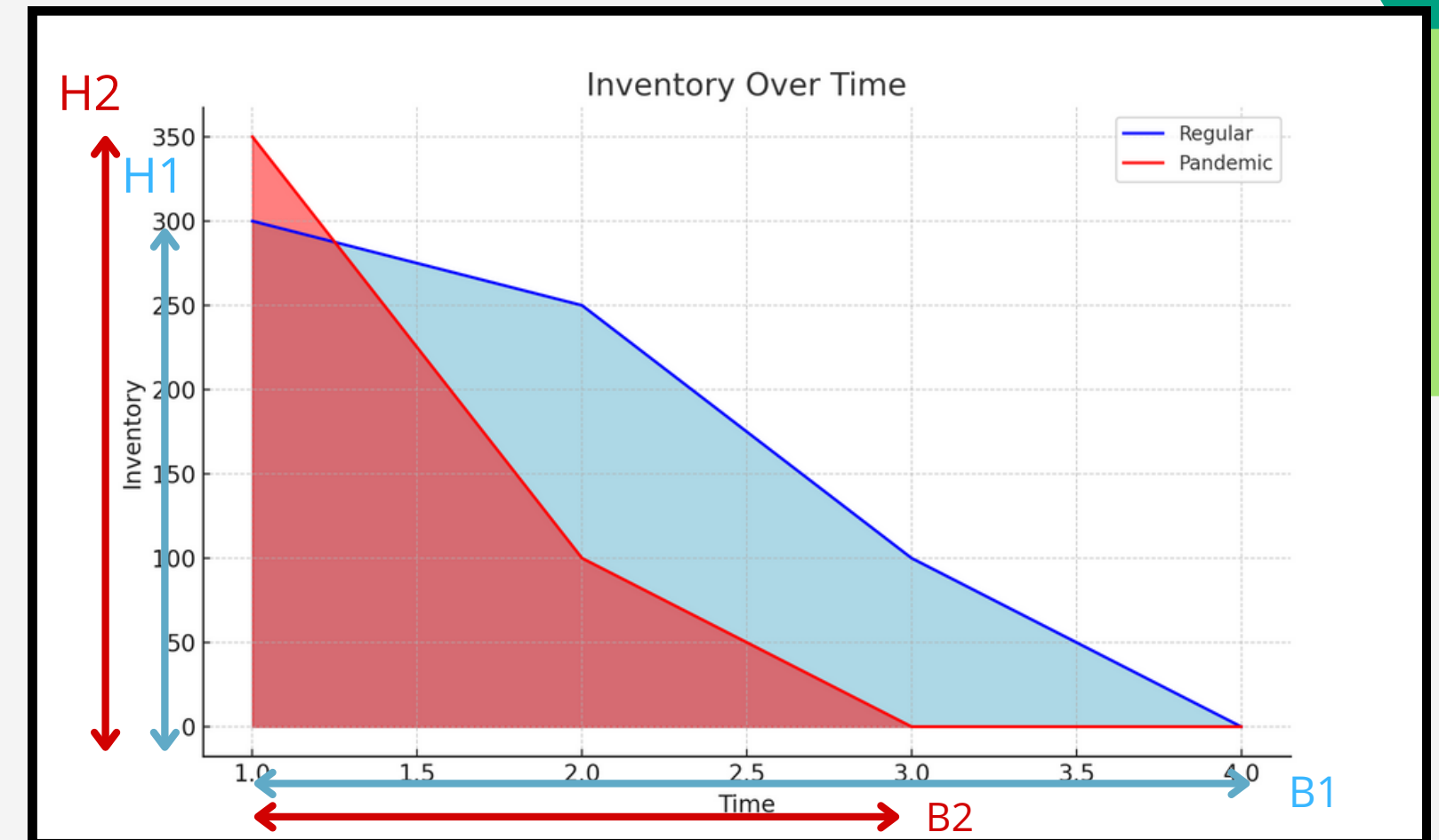
Why opt for two marginally distinct policies?

- Supermarkets prioritize Item Fill Rate because it **directly affects customer experience** and sales at an item level, crucial for retail environments with **direct customer interaction**.
- On the other hand, distribution centers prioritize Cycle Service Level as it impacts their ability to efficiently supply multiple retailers or endpoints with complete orders, reflecting the **overall reliability and efficiency** of the supply chain operations. Each metric aligns with the core objectives and operational needs of the respective environments.

Demand growth estimation

Store agents are unable to determine the exact quantity each customer intends to purchase upon entering the store; instead, they monitor the changes in inventory levels over time, which reflect customer purchases. In reality, we only have access to the inventory levels at the start of each day.

Given this context, to more accurately approximate demand growth over a restocking period, we propose an **original approach** calculating the ratio of the average inventory level during a restocking cycle (illustrated by the blue area) to the inventory level during a cycle in the pandemic period (illustrated by the red area), as the latter typically decreases more sharply. This ratio acts as an indicator of demand growth.



Demand growth estimation

The foundation of this ratio lies in understanding that average demand correlates with the negative slope of the inventory trajectory. Analytically, this demand (D) can be expressed as:

$$D = -\tan(\beta)$$

with β representing the angle between the inventory curve and the vertical axis.

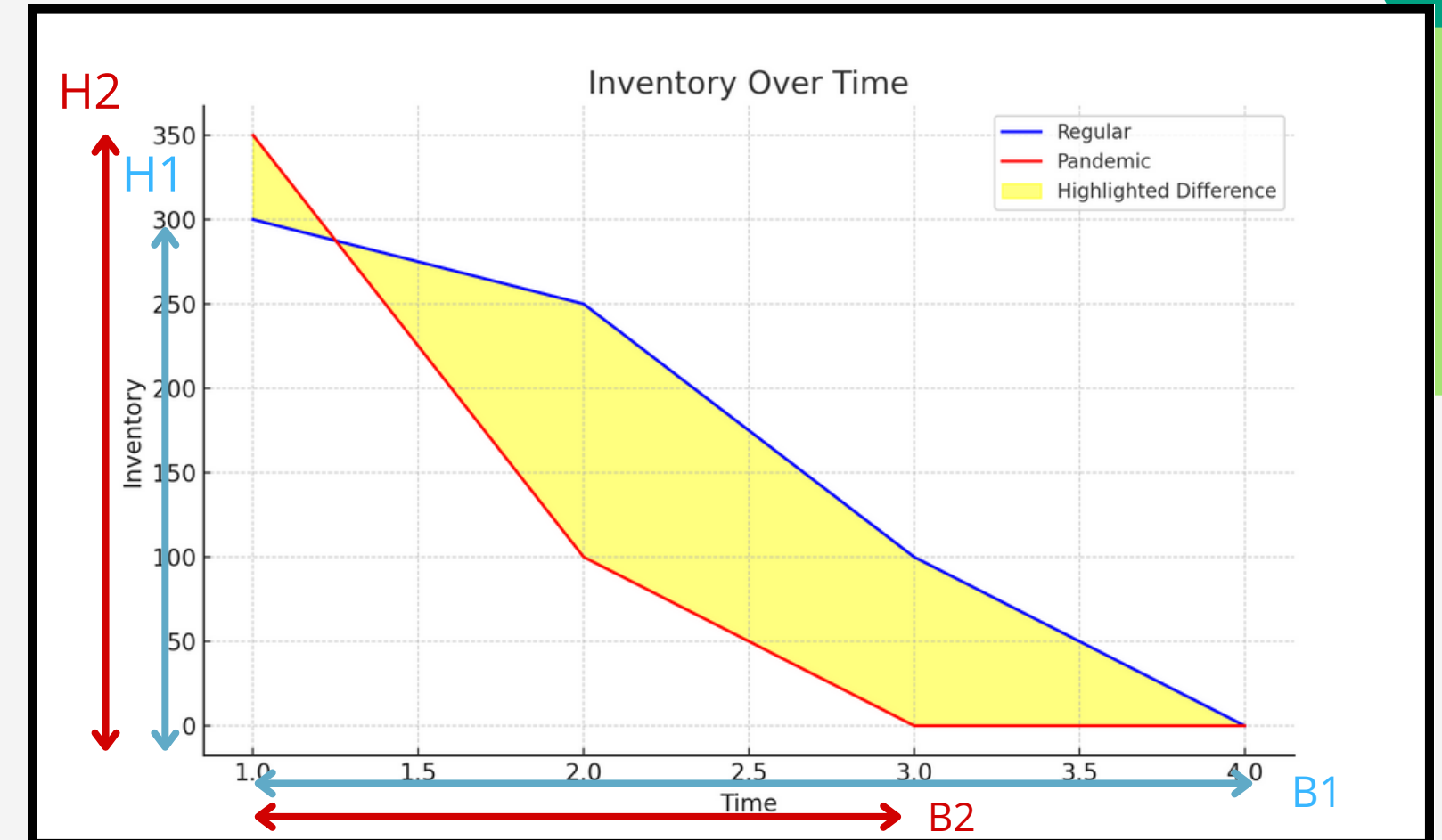
Expanding on this concept, we have enhanced our method to formulate a reliable indicator of demand, utilizing two readily obtainable metrics: the inventory level at the beginning of restocking (H) and the total area under the inventory trajectory. This improved approach provides a more sophisticated and readily implementable insight into the fluctuations of demand.

As a result, this ratio effectively represents the yellow area in the graph, serving as an indirect measure of demand growth.

$$\frac{D_1}{D_2} = \frac{-\tan(\beta_1)}{-\tan(\beta_2)} = \frac{B_1/H_1}{B_2/H_2} = \frac{H_2 \times B_1}{H_1 \times B_2} \times \frac{H_2}{H_2} \times \frac{H_1}{H_1} = \frac{Area_1}{Area_2} \times \frac{H_2^2}{H_1^2}$$

Upon calculating the ratio, it serves as a multiplier of the quantity demanded during a normal period **to deduce the quantity demanded during a pandemic** period, thereby enabling the placement of more accurate orders based on this adjusted demand.

$$D_{pandemic} = ratio \times D_{standard}$$



Costs analysis

- **Under increased panic buying scenarios**, the Fixed Reorder Period with Target availability model shows an interesting pattern. The item fill rate (IFR) is notably higher in the initial model than in our approach, mainly because the original model does not consider cost limitations. It aims to meet demand at all times, leading to higher costs during demand spikes.
- The **costs of implementing** an ICT system for continuous demand monitoring and predictions are substantial. Therefore, the (R,S) policy becomes essential in reducing and simplifying these costs.

Parameters	Meaning	Example Value
A	Order launch cost	80 €/order (best case) 100 €/order (base case)
v	Item value	1 €/item

$$C_{tot} = C_r + C_m + C_s$$

Cr: restocking costs
Cm: stocking costs
Cs: stock out costs

In case of **lost sales** (meaning that the stock outs are not going to be satisfied the next time the order arrives, but we consider them as lost demand).

Base Case

$$\left. \begin{aligned} C_r &= \frac{A}{R} = 33.3 \text{ €/cycle} \\ C_m &= \frac{D \times R}{2} = 600 \text{ €/cycle} \\ C_s &= D_{lost} \times v = 1350.5 \text{ €/cycle} \end{aligned} \right\} C_{tot} = \mathbf{1983.8 \text{ €}}$$

Modified Policy

$$\left. \begin{aligned} C_r &= \frac{A}{R} = 26.7 \text{ €/cycle} \\ C_m &= \frac{D \times R}{2} + ss = 650 \text{ €/cycle} \\ C_s &\approx 0 \end{aligned} \right\} C_{tot} = \mathbf{676.7 \text{ €}}$$

The difference in costs is substantial, in fact we have **cut 65.8 % of the cost** of the previous policy (NB: this estimation is done in normal conditions).

Pre-Simulation

The beginning stage of the simulation was focused on calculating the essential parameters required for the simulation's main phase. This method was adopted to mimic the **historical demand patterns**.

In the initial scenario, each store's initial inventory is determined merely by the demand it encounters on the simulation's opening day. This approach, however, is unlikely to provide an accurate reflection of the specific demand for each store.

Pre-Simulation Parameters

The preliminary parameters for the pre-simulation phase were determined based on the customer purchase patterns that form the foundation of the project. From this data, the average daily demand and its variability were assessed.

This step was crucial for establishing the initial **manufacturing capacity** and the **inventory** at the **Distribution Center** (DC), following the previously outlined policy. The same principle was applied to the stores, setting a **target inventory** equal for all of them during this initial phase.

After the pre-simulation, these parameters will be recalculated using the demand data gathered during this phase. The purpose of this recalibration is to ensure that each agent's inventory level is tailored to its specific demand.

Panic Buying Dynamics

Base Case Model

Static Panic Percentage

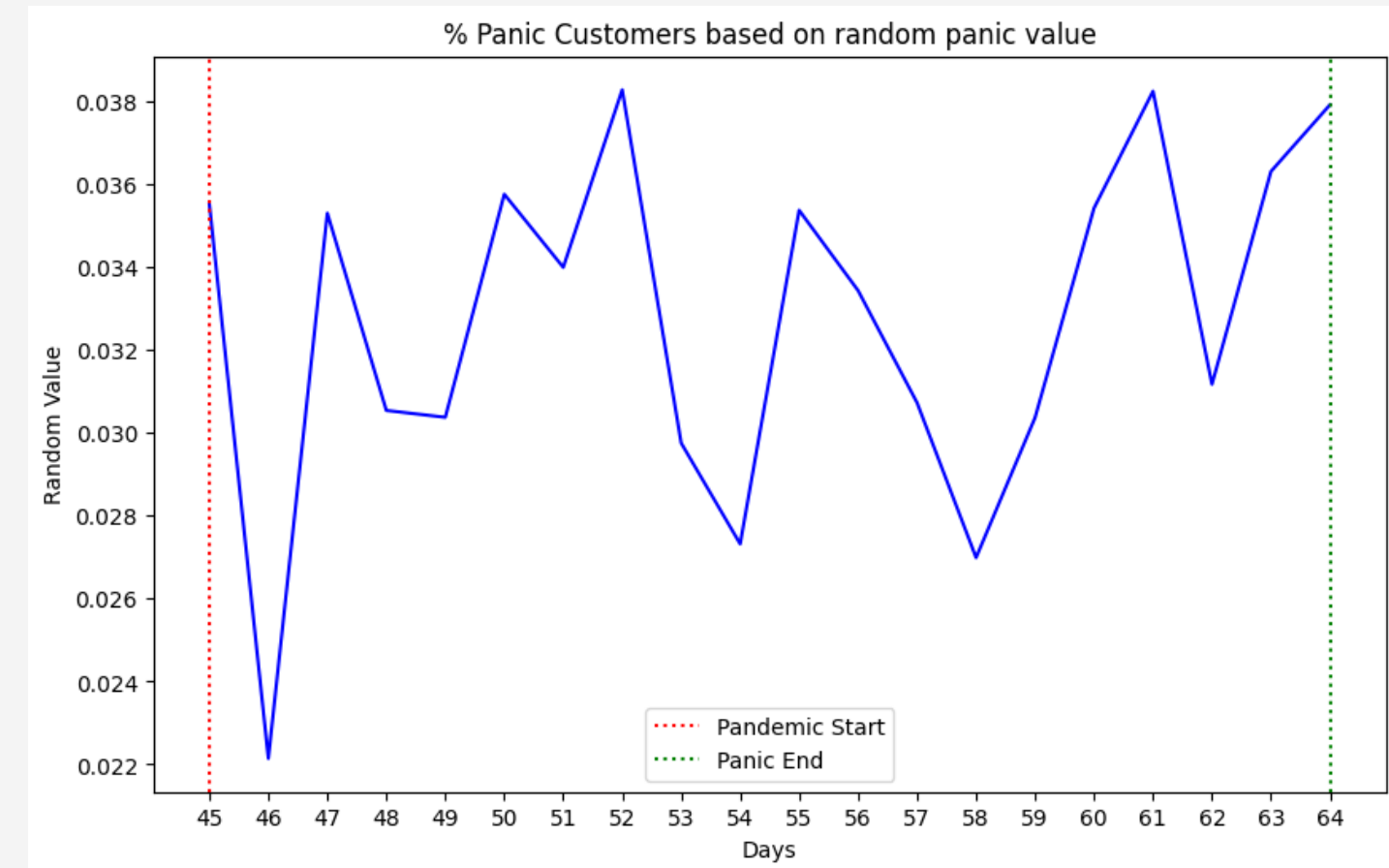
```
alpha_pbs = random.uniform(0.02, 0.04)
```

- The base case sets a fixed range for the percentage of panic buyers (2% to 4%), without accounting for the dynamic nature of a pandemic.
- This static approach means the model doesn't simulate changes in consumer behavior as the pandemic evolves.

Equal Distribution Across Regions

- This base case evenly distributes panic buyers across different regions. It divides the total number of panic buyers N_{pb} by the number of regions $N_{regions}$, then assigns an equal proportion of panic buyers to each region.
- This approach does not reflect the potential for varying panic levels in different regions based on factors like local news, government policies, or the spread of the pandemic.

```
if len(self.initial_panic_customers) < (N_pb*(w+1))/N_regions:  
    self.initial_panic_customers = np.append(self.initial_panic_customers, [a])  
self.customer_list_region.remove(a)
```



Panic Buying Dynamics

Modified Model

Dynamic Panic Levels

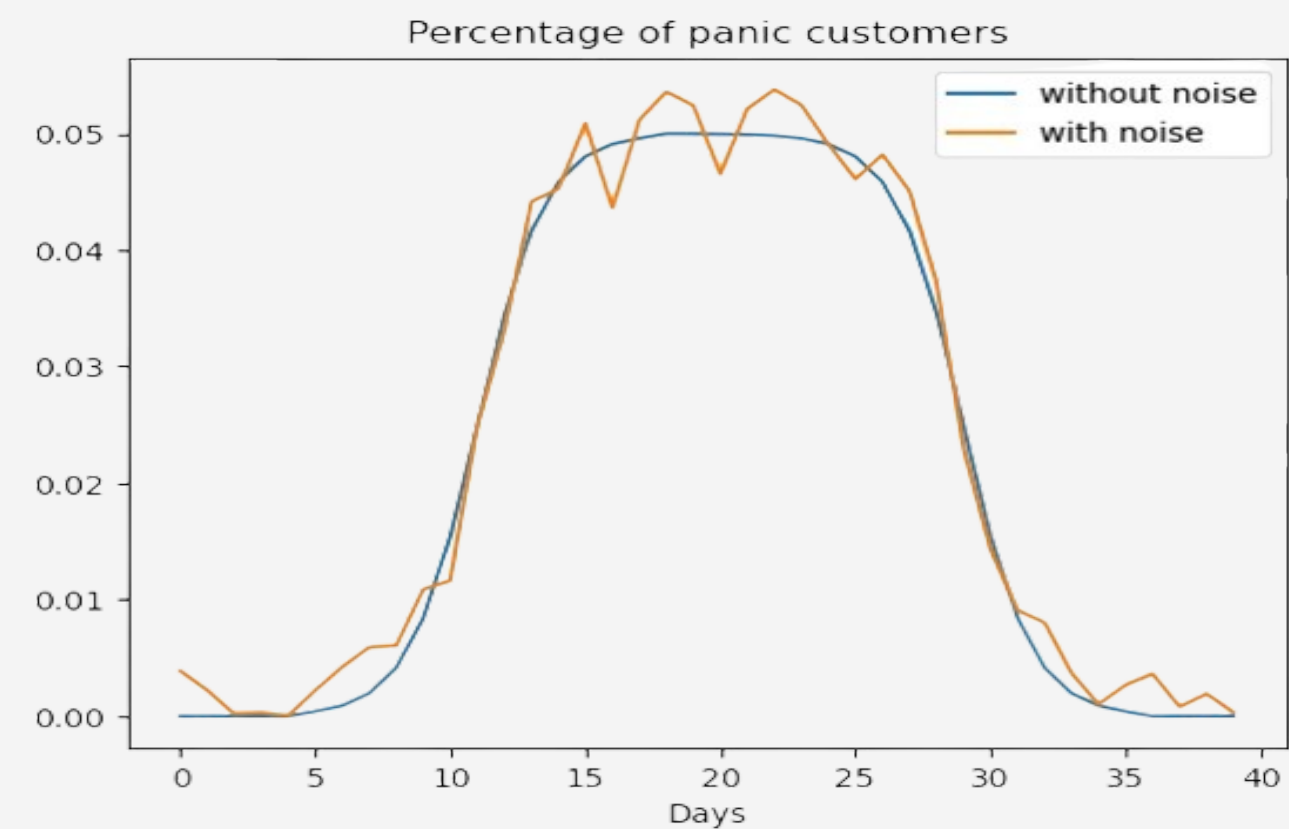
- This code segment dynamically calculates the percentage of customers engaging in panic buying α_{pbs} based on the current time step i relative to the pandemic's timeline.
- It uses logistic growth and decay functions to represent the onset and end of panic buying behavior.
- The use of logistic functions allows for a more realistic, gradual change in panic behavior over time, reflecting the evolution of a pandemic scenario.

Transition Phases

- The variable i represents the current time step, and the transition phases are defined by the $pandemic_start$, $pandemic_end$, and transition variables.
- The logistic functions vary the panic level α_{pbs} smoothly during these transition phases.

Random Noise Addition

- The noise is generated using a normal distribution centered around zero with a standard deviation defined by $noise_sd$.
- This introduces variability and unpredictability, mimicking the natural variations in human behavior under panic conditions.
- The graph below shows the percentage of customers in panic with and without the presence of random noise.



Customer Movement

Base Case Model

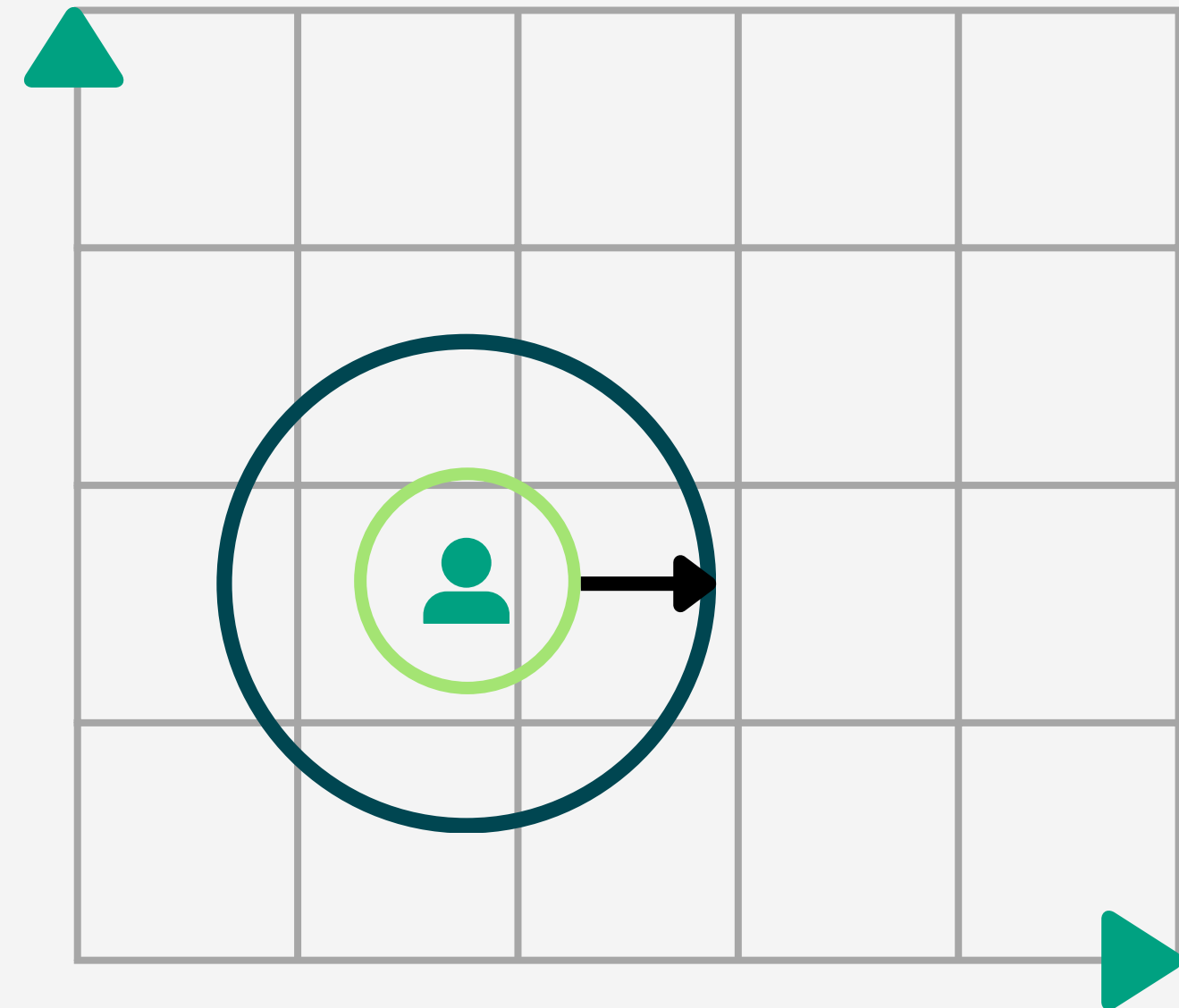
Limited Dynamics

Customers in the simulation select a store to visit randomly from their list of stores `self.stores`

This method of store selection does not consider the distance between the customer and the store. Instead, it treats all stores in the customer's list as equally likely choices, regardless of their location relative to the customer.

Modified Model

- Customers choose stores based on their distance, addressing the previous model's weakness of random store selection. In normal conditions, customers select stores within a specific radius `self.r`
- During panic, this radius is expanded `self.r*p_mult`, simulating increased effort to find supplies. This change introduces more realistic behavior, with customers preferring closer stores under normal circumstances and expanding their search area in panic situations.



*Illustration of Customer Reach in the new model:
Normal Conditions in **Light Green** and Panic
Conditions in **Blue***

DC-Waitlist

Base Case Model

In the base case, the Distribution Center (DC) handles store orders sequentially, fulfilling them as long as inventory units are available. Once the inventory is terminated, the remaining unfulfilled orders are canceled and registered as lost demand.

Item fill rate decrease over time

The waitlist's sequential approach, together with the initial low inventory level at the Distribution Center, results in a gradual decline in the item fill rate over time, even outside the pandemic period. This is because the DC consistently fulfills orders only from the first few stores of the list, which tend to be the same ones each time.

Consequently, customers gradually shift their shopping to the few stores that have items in stock. As a result, the demand at these stores grows over time, while the other stores, despite placing consistent orders, receive nothing. This situation leads to an overall increase in the orders placed to the DC.

Order Scheduling

It's important to note that all stores place their orders simultaneously, resulting in the distribution center (DC) needing to process every restock order on the same day. In practice, this could pose a significant challenge for the DC, as managing the surge of orders concentrated into a single day would be demanding. A more practical approach in a real-world setting would be to spread out the store orders over three days to distribute the workload more evenly.

DC-Waitlist

Modified Model

In the modified file, the waitlist has been modeled following a different approach. In fact, this time the DC before fulfilling the orders of the stores, computes a ratio that is calculated as:

$$\left\{ \begin{array}{ll} 1 & \text{if DC inventory} > \text{total stores demand} \\ \frac{\text{DC inventory}}{\text{tot stores demand}} & \text{if DC inventory} < \text{total stores demand} \end{array} \right.$$

Balanced Supply Distribution

- This way, when the availability of the DC is higher than the demand, it will satisfy only the requested quantities. However, in times of scarcity, every store will be guaranteed a minimum percentage of their order.
- This strategy prevents situations where certain stores receive nothing, a problem that was evident with the former method.

Tailored Supply Allocation

The decision to calculate this percentage rather than allocate identical stock quantities to every store was made in recognition of the varying demand sizes among different stores. This variation is primarily due to the updated method of customer movements, where households limit their shopping to stores that fall within a specific radius. Consequently, there is a greater likelihood that stores situated in areas with fewer surrounding customers, such as grid corners, will experience reduced demand.

Modified Model Performance

Performing a comparative analysis of our updated model against the previous version. Focused on enhancing realism in simulating panic buying scenarios, the modified model reveals significant improvements in the performance metrics.

Demand Variation over time

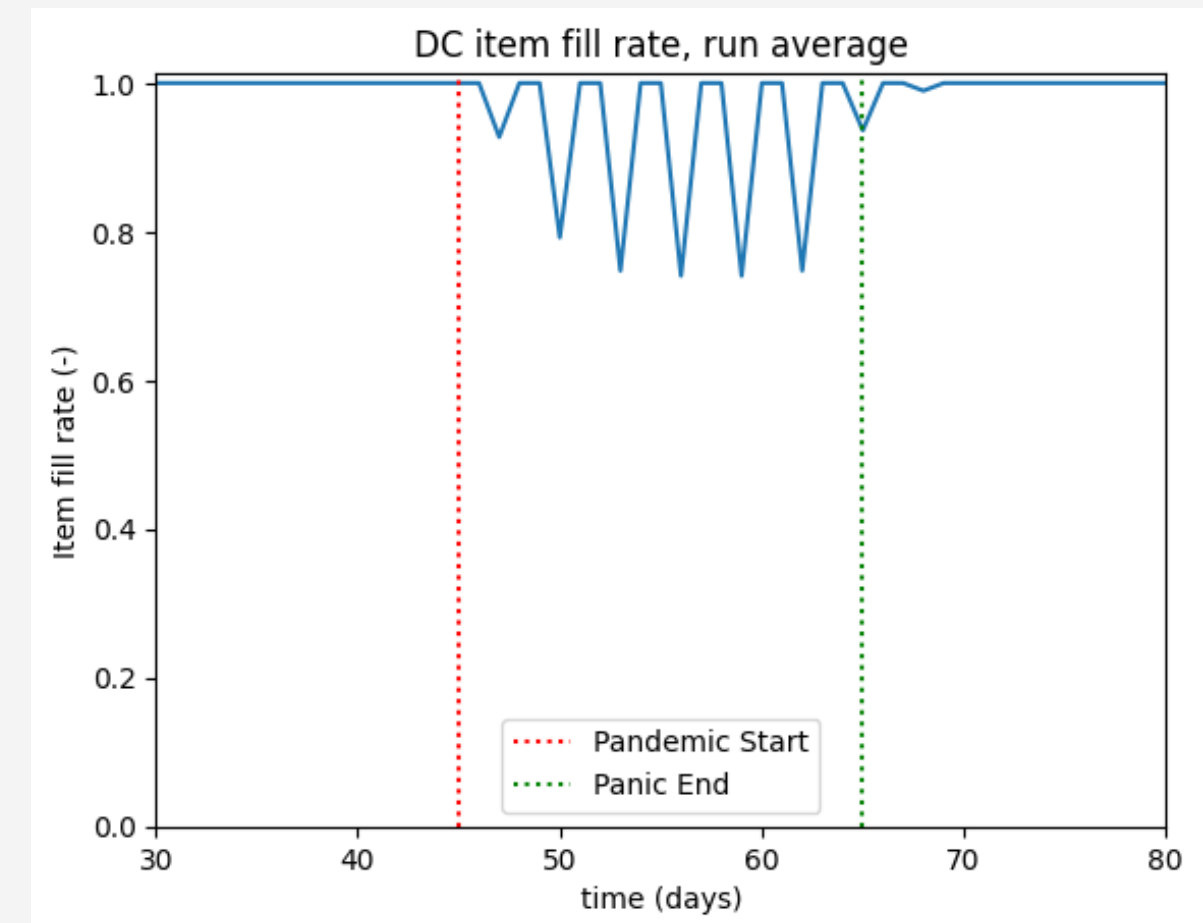
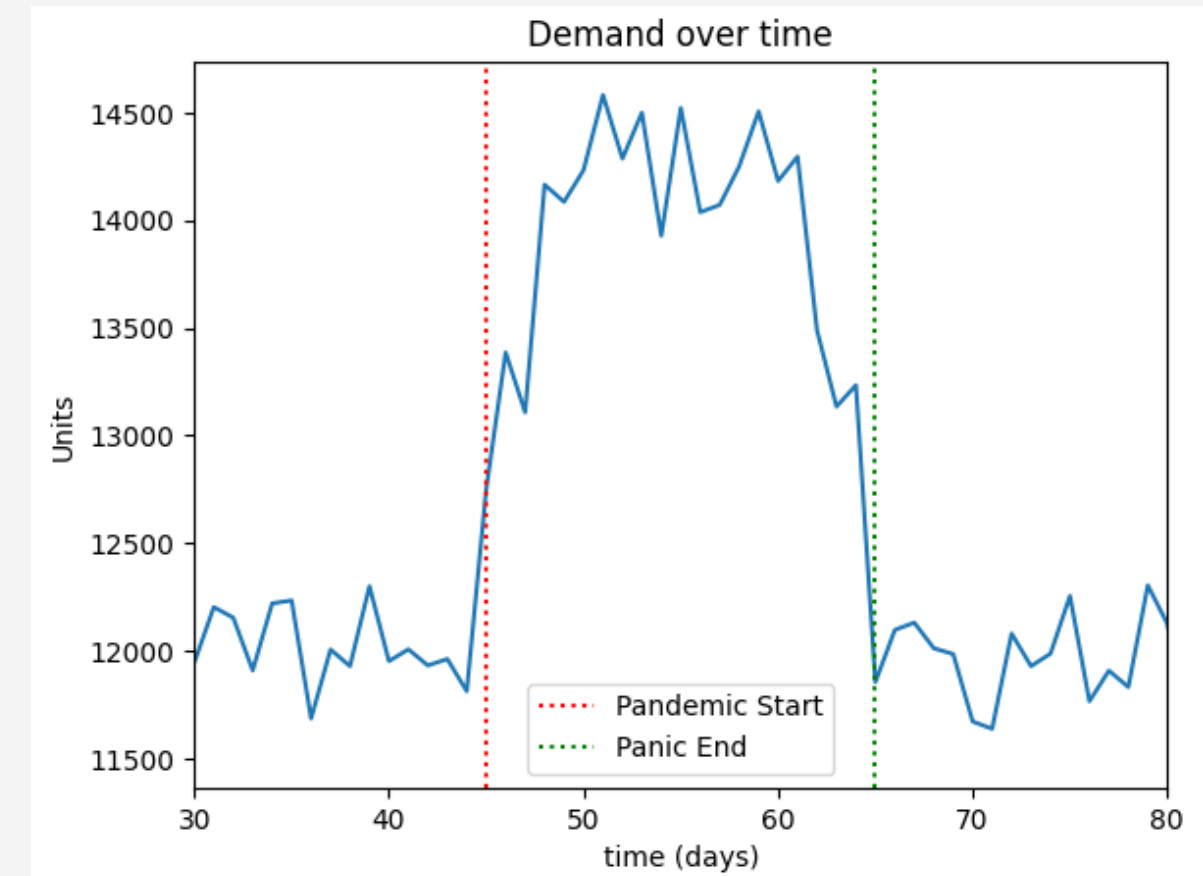
- The modified model shows a steeper demand peak at the pandemic's onset, signaling an intensified consumer response.
- Demand spikes to a higher maximum in the new model, suggesting a stronger and more immediate panic buying reaction.
- Panic buying in the updated model is brief with defined start and end points, closely mirroring actual consumer patterns in pandemic situations.

Item Fill Rates

Minimum item fill rate in normal period: **98.97 %**

Minimum item fill rate during pandemic: **74.07 %**

- While the old model exhibited minimal variations between normal and pandemic periods (**12.74% vs. 13.44%**), the new model displays a significant change, with **98.97%** in normal conditions dropping to **74.07%** during the pandemic.
- By showing a substantial difference in item fill rates between normal conditions and pandemic periods, it better reflects the complex dynamics of supply chain, providing more accurate and insightful analysis.



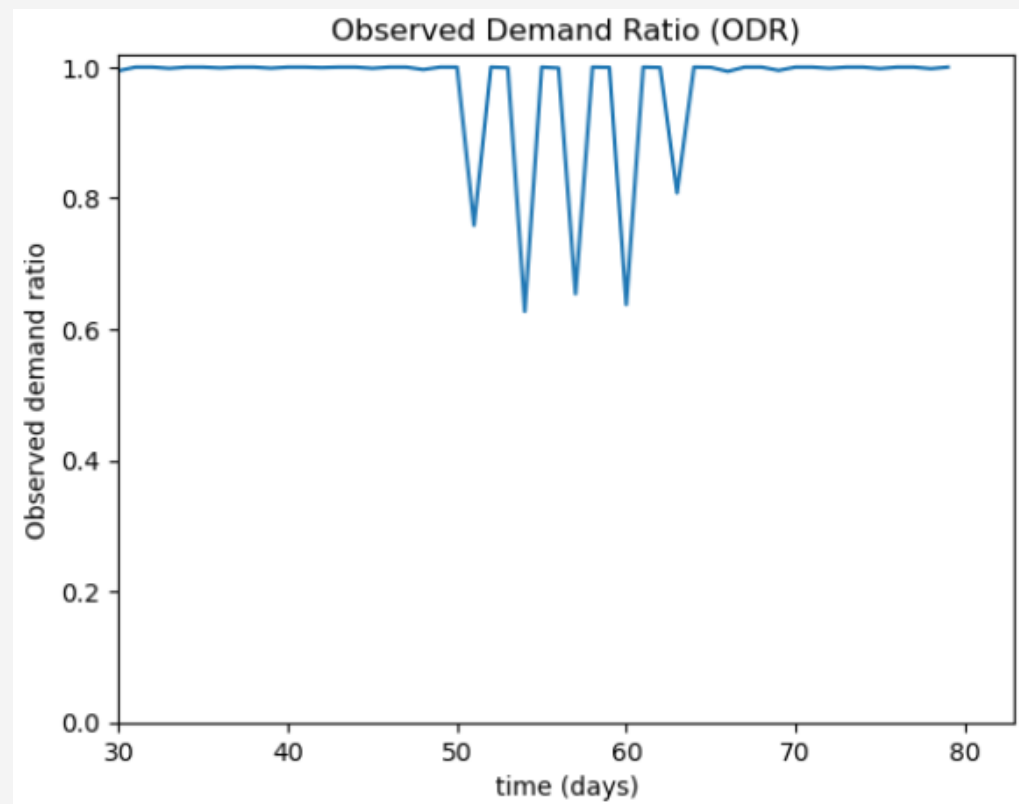
New Performance Measure

Unobserved demand by the stores

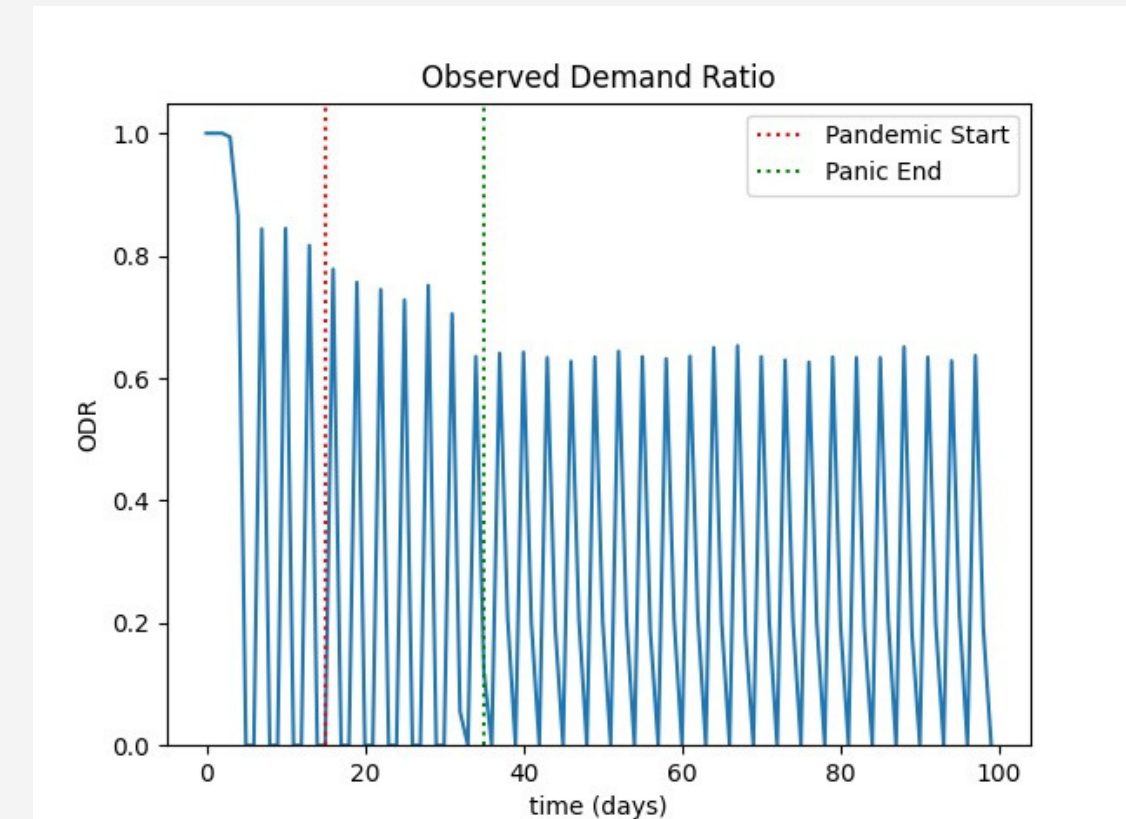
The **Observed Demand Ratio** refers to the portion of customer demand that remains invisible to stores because they calculate pasta demand based on the quantity purchased, rather than the quantity customers originally intend to buy upon entering the store.

The indicator is computed with the following formula:

$$ODR = \frac{RealDemand - StoreDemand}{RealDemand}$$



Observed demand ratio for the modified case



Observed demand ratio for the base case

- The two charts compare the ratios for both models, highlighting the advancements of the newer one. This metric is key for evaluating how well the stores meet actual demand, serving as an important performance indicator.
- Its significance was amplified during the pandemic analysis; an incorrectly applied policy indicated an IFR of 1 throughout this period. However, by integrating this metric, we could detect the discrepancy.
- It's worth noting that individual stores cannot calculate this indicator due to the unknown quantity of unmet demand, but it serves as a valuable KPI for analyzing the network's collective efficiency.

Business Insights

Optimized Transportation Logistics in Fixed Reorder Period

Strategic Scheduling: Establishing consistent transportation days with upstream partners reduces emergency shipping needs and associated costs.

Lead Time Stability: Regular scheduling enhances upstream efficiency, ensuring more reliable and faster deliveries.

Consolidated Shipping for Multiple Items

Cost Efficiency: Thanks to planned delivery days it is easier to aggregate shipments for various items (e.g., pasta, cereal, milk) into a single order substantially cutting down the transportation expenses.

Target Inventory Availability Strategy

Predictive Analytics: In highly stable demand scenarios where neither seasonality nor trends are present, frequent demand prediction is unnecessary, lowering implementation costs and minimizing forecasting errors.

Adaptive Policy Management

Dynamic Strategy: Recognizing the need for flexibility in policy application, depending on evolving market conditions, and data-driven decision-making. In fact does not exist a policy that fits perfectly all the scenarios.

Support Systems: Implementation of alert systems, simulations, and other Information and Communication Technology (ICT) tools for effective policy adjustments.

Appendix

References: 'Gestione delle scorte- Modelli matematici e loro applicazione' by Lorenzo Tiacci, Second Edition

ESPRC ('Expected Stock-out Per Replenishment Cycle'):

It is the expected value of the quantity $(x-s)$, which represents how much the demand during lead time is exceeding the reorder point s .

$$ESPRC = \int_s^{\infty} (x - s) \cdot f_L(x) dx$$

We can assume a Gaussian distribution for the demand

$$ESPRC = \int_{\hat{x}_L + k\sigma_L}^{\infty} (x - \hat{x}_L - k\sigma_L) \cdot \frac{1}{\sigma_L\sqrt{2\pi}} e^{-\frac{(x - \hat{x}_L)^2}{2\sigma_L^2}} dx$$

Substituting $u = \frac{x - \bar{x}_L}{\sigma_L}$:

$$ESPRC = \sigma_L \int_k^{\infty} (u - k) \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du = \sigma_L G_u(k)$$

Given the Cumulative Distribution function $F_u(x)$ of the Gaussian:

$$F_u(x) = \int_x^{\infty} f_u(u) du$$

We can define the Probability of not going in Stock-out in a cycle as:

$$P_u(x) = 1 - F_u(x) = \int_x^{\infty} f_u(u) du$$

